Analytic Design of Digital Flight Controllers to Realize Aircraft Flying Quality Specifications

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This paper treats the problem of meeting given flying quality specifications for aircraft using a digital control system. First, a model of an aircraft that possesses the desired flying qualities is obtained using a continuous time control system design process. Then an optimal digital controller is designed so that the responses of the digital controlled aircraft correspond, as closely as possible, to those of the model. Results are presented for a lifting body entry vehicle. For typical sampling rates (say about 0.1 sec. for aircraft) the digital feedback gains and control crossfeeds are shown to differ substantially from those of the model. The results also show that the responses of the digitally controlled vehicle using the model feedback gains and crossfeeds—uncorrected for the discrete nature of the controller—may lead to system instability. However, for properly designed digital control systems, with computation times much shorter than the system cycling time, the digital controlled system follows the model well even for cycling times as high as 1 sec.

Nomenclature

A = $n \times n$ state matrix of $\dot{x}$ equations
C = $n \times m$ input matrix
F = $m \times m$ continuous-time control crossfeed matrix
G = $m \times n$ continuous-time gain matrix
I_k = $k \times k$ identity matrix
J = quadratic form of Δx_{k+1}
k, m, n = integers
p = roll rate
Q = positive definite symmetric matrix
r = yaw rate
t, t_0 = time
u = $m \times 1$ unaugmented input vector
u_p = $m \times 1$ pilot input vector
v_k = $m \times 1$ vector equal to u_{k-1}
x = $n \times 1$ state vector
z = z-transform variable
β = sideslip angle, deg
$\Gamma(\tau)$ = $n \times m$ discrete control input matrix for τ
$\bar{\Gamma}$ = $(n+m) \times m$ matrix defined in Eq. (18)
$\nabla_{u_k} J$ = $m \times 1$ vector whose ith element is $\partial J/\partial(u_{k_i})$
Δx_{k+1} = $x_{k+1} - x_{d_{k+1}}$
δ_a = aileron deflection, deg
δ_r = rudder deflection, deg
θ = digital controller computation delay
σ = dummy variable of integration
τ = digital controller cycling time
$\Phi(\tau)$ = $n \times n$ discrete transition matrix for τ
$\bar{\Phi}$ = $(n+m) \times (n+m)$ matrix defined in Eq. (18)
ϕ = angle of bank, deg

Subscripts

d = indicates desired characteristic
k = indicates evaluation at $t = k\tau$

Introduction

DIGITAL flight control systems offer many advantages over continuous-time control systems. They provide a flexibility not possessed by continuous-time systems and offer advantages in weight saving and simplicity of design. However, they have not been used in service for the complete navigation and control task due to component cost, reliability, and the lack of fly-by-wire technology. These problems are being resolved and complete digital flight control systems will probably be in use in some future aircraft designs.

Both discrete-time digital and continuous-time automatic flight control systems are used on aircraft to make the closed-loop response characteristics of the vehicle satisfy flying quality requirements. These requirements have resulted from many years of flight experience. They vary for different aircraft categories, and are continually subject to revision.[1] Also, they are generally well suited for design of continuous-time control systems for aircraft because many flying quality specifications can be algebraically related to the feedback gains and control crossfeeds of linear feedback control systems. These relationships have been exploited in Ref. 2 where a process for designing continuous-time control systems for aircraft to meet prespecified flying quality specifications was developed.

Techniques for designing discrete-time digital control systems include z-transform methods (analogous to Laplace transform methods in continuous-time systems) optimal regulator methods, and deadbeat design methods (for which no appropriate analogy exists in continuous-time systems). These methods are intended to realize a digital controller which satisfies design requirements stated in terms of discrete-time control system specifications. A major problem in the design of digital flight control systems, however, is that design requirements for aircraft are not easily translated into discrete-time control system specifications.

The purpose of this paper is to describe a method applicable for a single flight condition for the design of discrete-time digital control systems for aircraft that will result in closed-loop aircraft response characteristics that approximate, to any prescribed degree of precision, arbitrarily prescribed flying quality specifications as specified in documents such as Ref. 1. This is accomplished using a two-step design process. First, a continuous-time control system is designed, using the techniques of Ref. 2, to provide a model of aircraft responses that the prescribed flying quality specifications. Then, a digital flight control system is designed, using optimal control theory, so that the responses of the digitally controlled aircraft follow, as closely as possible, those of the model.

Analytic Development

General Considerations

The first step in the design process described herein is to design a continuous-time control system with the desired flying qualities using the procedure developed in Ref. 2. Although

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it is possible to construct a model of the aircraft dynamics without resorting to a continuous-time design process, it is not clear that the actual aircraft will be able to follow the responses of a model so derived. Following the procedure of Ref. 2 assures that the model will be compatible with the actual aircraft. Next, the dynamic equations of motion of the aircraft and those of the model are put into discrete form for a given digital system cycling time. Finally, a digital controller is determined, using optimal control theory, that minimizes, at each sampling instant, the error between the predicted response of the actual aircraft and the response of the model at the next sampling instant.

Difference Equation

The purpose of this section is to derive difference equations that may be used to calculate the evolution of the dynamical system under the action of a "staircase" control function that is based on sampling and digital computation at even intervals of time. The equations of motion of the unaugmented aircraft can be linearized and put in the form

$$\dot{x} = Ax + Cu \tag{1}$$

This form is the same one used in Ref. 2. Here, x is an n-dimensional vector representing the state of the aircraft, u is an m-dimensional vector representing the controls of the aircraft, A is an $n \times n$ matrix obtained from the aircraft's mass properties and stability derivatives, and C is an $n \times m$ matrix determined from the aircraft's mass properties and control effectivenesses. In general, the solution to the equations of motion is expressible as

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\delta)}Cu(\sigma)d\sigma \tag{2}$$

where t_0 is a time at which the state is known, the initial conditions, and $u(\sigma)$ is the control applied to the aircraft during the time interval $t_0 \leqq \sigma \leqq t$. In Eq. (2) the term e^{At} represents the infinite matrix series

$$I_n + At + \frac{(At)^2}{2!} + \cdots + \frac{(At)^k}{k!} + \cdots \tag{3}$$

Equation (2) can provide time histories of the state variables $x(t)$ for arbitrary inputs $u(t)$. For digital systems the state is sampled at various times and, based on these discrete samples, a control input can be determined by a digital computer. This input is discrete, a series of numbers, and must be applied to the aircraft in some sectionally continuous form. A zero-order hold is a popular and convenient way to convert the discrete computer commands into the actual control function. This is the process used in this paper, and it is illustrated in Fig. 1. The state x varies continuously as illustrated at the top of the figure and is sampled every τ units of time. The control u is determined only at the sampling instants and held constant throughout the controller cycle. There is normally a delay time θ between the time that measurements are taken and the time that the control is determined and applied to the vehicle. This delay is considered later but is zero for the present development.

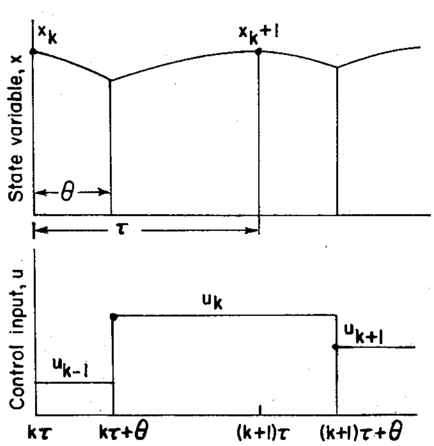


Fig. 1 A typical state variable time history using digital control including computation delay.

In designing digital control systems, it is more convenient to work with difference equations rather than with either Eqs. (1 or 2). To obtain the difference equations, assume a cycling time τ and let x_k be the state vector $x(t)$ when $t = k\tau$. Since a zero-order hold is used, set

$$u(t) = u_k \qquad (k-1)\tau \leqq t < k\tau$$

This is consistent with Fig. 1. The difference equation governing the aircraft's motion can now be obtained from Eq. (2) by setting $t = (k+1)\tau$ and $t_0 = k\tau$. The result is

$$x_{k+1} = \Phi(\tau)x_k + \Gamma(\tau)u_k \tag{4}$$

where

$$\Phi(\tau) = e^{A\tau}$$

$$\Gamma(\tau) = \int_0^{\tau} e^{A\sigma}d\sigma C \tag{5}$$

Note that if some form of control application other than zero-order hold is used, he only term affected is the matrix Γ.

The Model

The linear continuous-time controller may be designed to satisfy a variety of flying quality specifications. Its determination is the most important step in the design process used here. This matter has been treated in detail in Ref. 2, however, and will not be covered here. The result of the analysis is a control system of the form

$$u = Gx + Fu_p \tag{6}$$

where G is an $m \times n$ feedback gain matrix, F is an $m \times m$ control interconnect matrix, and u_p is an m-dimensional pilot input vector.

When the linear continuous-time controller is used, the resulting behavior of the aircraft's state satisfies the closed-loop equation

$$\dot{x} = (A + CG)x + CFu_p \tag{7}$$

By direct analogy with the results of Eq. (4) the discrete form of the compatible augmented aircraft is

$$x_{d_{k+1}} = \Phi_d x_{d_k} + \Gamma_d u_{p_k} \tag{8}$$

where the subscript "d" is used to distinguish the "desired" state of the aircraft (that is, that of the model) from the state of the actual vehicle which behaves according to Eq. (4). In Eq. (8) the matrices Φ_d and Γ_d are given by

$$\Phi_d = e^{(A+CG)\tau}$$

$$\Gamma_d = \int_0^{\tau} e^{(A+CG)\sigma}d\sigma\, CF \tag{9}$$

Since the inputs to the vehicle are piecewise constant, Eq. (8) yields the precise output of the model.

Optimal Digital Controller

The criterion used in the design of the digital controller is to make the digitally controlled aircraft, governed by Eq. (4), behave, as closely as possible, like the desired compatible augmented aircraft, governed by Eq. (8). To do this, an analytic expression of the closeness of responses is needed.

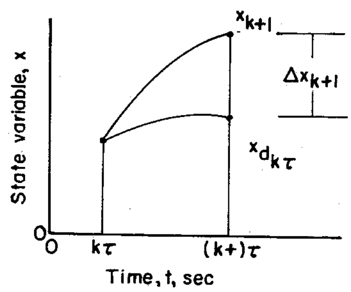


Fig. 2 Diagram illustrating the actual and desired aircraft responses over the time interval $k\tau \le t \le (k+1)\tau$.

Referring to Fig. 2, if the aircraft is in state x_k at $t = k\tau$ the application of control u_k over the time interval $k\tau \le t \le (k+1)\tau$ will lead the aircraft to the state x_{k+1} at $t = (k+1)\tau$. Assuming the same conditions for the desired augmented aircraft at $t = k\tau$ (i.e., $x_k = x_{d_k}$) the application of the pilot input u_{p_k} would lead the aircraft to state $x_{d_{k+1}}$ at $t = (k+1)\tau$. The two states, x_{k+1} and $x_{d_{k+1}}$, generally differ by an amount Δx_{k+1} which, given the pilot input u_{p_k}, depends on the control u_k. The closeness of the responses can, therefore, be measured by $\Delta x_{k+1} = x_{k+1} - x_{d_{k+1}}$. Hence, analytically, the digital controller is designed for a piloted input so that, given the state x_k, the control u_k makes the quadratic form

$$J = \Delta x_{k+1}{}^T Q \Delta x_{k+1} \tag{10}$$

as small as possible.

The minimization of the function J can be accomplished by differentiation of J with respect to u_k, setting the resulting derivative, $\nabla u_k J$, to zero, and solving the resulting expression for u_k. Using Eqs. (4) and (8), the derivative of J with respect to u_k is given by

$$\nabla_{u_k} J = 2\Gamma^T(\tau) Q[(\Phi(\tau) - \Phi_d)x_k + \Gamma(\tau)u_k - \Gamma_d u_{p_k}] \tag{11}$$

Setting $\nabla u_k J$ equal to zero and solving for u_k results in

$$u_k = Hx_k + Eu_{p_k} \tag{12}$$

where

$$\left.\begin{aligned} H &= [\Gamma^T(\tau)Q\Gamma(\tau)]^{-1}\Gamma^T(\tau)Q[\Phi_d - \Phi(\tau)] \\ E &= [\Gamma^T(\tau)Q\Gamma(\tau)]^{-1}\Gamma^T(\tau)Q\Gamma_d \end{aligned}\right\} \tag{13}$$

The digital control system given by Eq. (12) is of the same form as that of the continuous-time system [Eq. (6)]. Since the model was derived using a continuous-time design process, the matrices H and E approach G and F, respectively, as $\tau \to 0$. Although feedback of the entire state is not necessary for the continuous-time controller, Eq. (6), it is required in the digital controller design, Eq. (12).

Optimal Digital Controller with Computational Delay

In all digital systems there is a delay between the time that measurements are taken and the time that the appropriate control is determined. In the design process used here, the primary item affected by the delay is the form of the discrete equations of motion of the vehicle. Consider the controller cycle to start at time $k\tau$ as indicated on Fig. 1. Measurements are taken at time $k\tau$. From these, the new control position u_k is determined. However, the process of determining and setting the control requires θ units of time. Hence, the old control u_{k-1} remains applied to the vehicle from time $k\tau$ until time $k\tau + \theta$. The state of the vehicle at time $(k+1)\tau$ will, therefore, be influenced not only by the control u_k as in Eq. (4) but also by u_{k-1}. From Eq. (2), the state at time $k\tau + \theta$ can be determined.

$$x(k\tau + \theta) = \Phi(\theta)x(k\tau) + \Gamma(\theta)u_{k-1}$$

Also, using the same procedure

$$x_{k+1} = \Phi(\tau - \theta)x(k\tau + \theta) + \Gamma(\tau - \theta)u_k$$

Combining the last two equations gives

$$x_{k+1} = \Phi(\tau - \theta)\Phi(\theta)x(k\tau) + \Phi(\tau - \theta)\Gamma(\theta)u_{k-1} + \Gamma(\tau - \theta)u_k$$

The following identities can be used to reduce x_{k+1} to a more convenient form

$$\Phi(\tau) \equiv \Phi(\tau - \theta)\Phi(\theta)$$
$$\Gamma(\tau) \equiv \Gamma(\tau - \theta) + \Phi(\tau - \theta)\Gamma(\theta)$$

which hold for any value of τ or θ. Hence

$$x_{k+1} = \Phi(\tau)x_k + [\Gamma(\tau) - \Gamma(\tau - \theta)]u_{k-1} + \Gamma(\tau - \theta)u_k \tag{14}$$

Equation (14) is the discrete form of the dynamics for a system using zero-order hold and accounting for computation delay. It will be used in the design process instead of Eq. (4). The object of the controller design is still the same—to make the vehicle's responses governed by Eq. (14) behave, as closely as possible, like the responses of the compatible model governed by Eq. (8). Again, closeness of response will be measured in the same way—using the function J given by Eq. (10). When Eq. (14) is used in J the control which minimizes J is given by:

$$u_k = Ku_{k-1} + Hx_k + Eu_{p_k} \tag{15}$$

where

$$\begin{aligned} k &= -T[\Gamma(\tau) - \Gamma(\tau - \theta)] \\ H &= T[\Phi_d - \Phi(\tau)] \\ E &= T\Gamma_d \\ T &= [\Gamma^T(\tau - \theta)Q\Gamma(\tau - \theta)]^{-1}\Gamma^T(\tau - \theta)Q \end{aligned} \tag{16}$$

Note that as $\theta \to 0$ the controller given by Eqs. (15) and (16) approaches the one given by Eqs. (12) and (13). Also note that the control law given by Eq. (15) produces additional modes of motion due to the u_{k-1} term that appears on the right-hand side of Eq. (15). By defining $v_k \triangleq u_{k-1}$ the closed-loop responses satisfy the difference equations

$$\begin{bmatrix} x_{k+1} \\ v_{k+1} \end{bmatrix} = \bar{\Phi} \begin{bmatrix} x_k \\ v_k \end{bmatrix} + \bar{\Gamma} u_{p_k} \tag{17}$$

where

$$\bar{\Phi} \triangleq \left[\begin{array}{c|c} \Phi(\tau) + \Gamma(\tau - \theta)H & \Gamma(\tau - \theta)K + \Gamma(\tau) - \Gamma(\tau - \theta) \\ \hline H & K \end{array}\right]$$
$$\bar{\Gamma} \triangleq \left[\begin{array}{c} \Gamma(\tau - \theta)E \\ \hline E \end{array}\right] \tag{18}$$

The characteristic equation of the closed-loop system is given by the equation

$$|zI_{n+m} - \bar{\Phi}| = 0 \tag{19}$$

where z is the z-transform variable. Zeros of Eq. (19) define the modes of the closed-loop system. As θ approaches zero the matrix $(zI_{n+m} - \bar{\Phi})$ approaches

$$\left[\begin{array}{c|c} zI_n - \Phi(\tau) - \Gamma(\tau)H & 0 \\ \hline H & zI_m \end{array}\right]$$

and since $\lim_{\theta \to 0} |zI - \bar{\Phi}| = |\lim_{\theta \to 0}(zI - \bar{\Phi})|$ the characteristic Eq. (19) becomes

$$z^m |zI_n - \Phi(\tau) - \Gamma(\tau)H| = 0 \tag{20}$$

in which case the system degenerates into one which has the same modes as if Eqs. (12) and (13) were used for the control, and, in addition, m other modes introduced by the controller at $z = 0$. As θ becomes larger than zero, the m controller modes move but the others remain fixed. This behavior will be illustrated for a specific example later.

Design Examples

Characteristics of Continuous-Time Control

The design procedure presented here for specification of flying quality requirements using digital flight controllers has been applied to the design of a digital system for the lateral control of the same vehicle used in Ref. 2. That vehicle was a lifting-body entry vehicle at an altitude of 60,000 ft and a Mach number of 1.8. The trim angle of attack was 15°. The unaugmented response of the vehicle to a step aileron input is illustrated in Fig. 3a. It is unacceptable due to the problems of roll reversal, induced sideslip, low damping in the roll subsidence and dutch roll modes, and strong coupling of the dutch roll mode in the roll response. This response can be improved by using any of several continuous-time control system designs presented in Ref. 2. From them, case 2 of Ref. 2 has been selected as the desired system for use here. When this control system is used, the aircraft responds to a step δ_{a_p} input as shown in Fig. 3b.

Characteristics of Digital Control

The digital design procedure has been used to determine the optional digital controller for cycling times ranging from 10^{-6} sec to 1 sec. Results presented in Tables 1 and 2 show the control system gains for no computation delay. These tables contain the optimal digital feedback gains and control crossfeed ratios for cycling time of 10^{-6} sec, 10^{-1} sec, and 1 sec. Since the aircraft and model are compatible, the feedback gains and crossfeed ratios of the optimal digital controller approach those of the analog controller as cycling time approaches zero. For moderate cycling times, however (say about 0.1 sec for aircraft), the optimal feedback gains and crossfeed ratios differ substantially from those of the analog controller.

Simulation

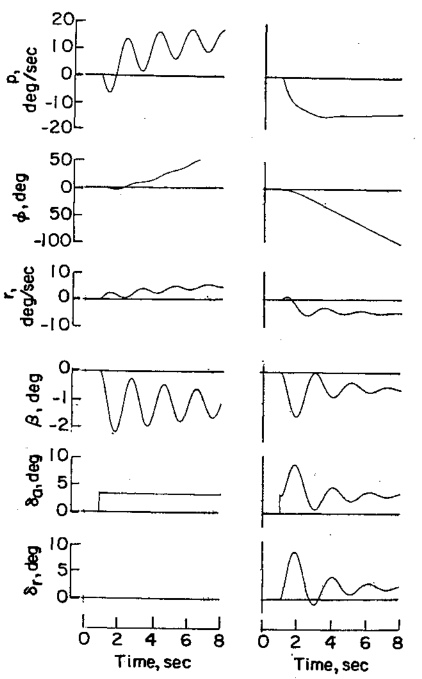
A simulation of the digitally controlled vehicle was undertaken to obtain working experience with digital control systems. The vehicle dynamics were programed on an EAI 680 analog computer and the digital control system was mechanized using an EAI 640 digital computer. The digital computer program allowed the computer cycling time and feedback gain and control crossfeed matrices to be read in as data. Results of the simulation are presented in Figs. 4 and 5. These figures contain responses of the vehicle to a step aileron input for computer cycling times of 0.1 sec and 1.0 sec, respectively. Each figure shows the response of a vehicle which is digitally controlled to a step aileron input for two cases. One is for the digital system using the continuous-time control system feedback gains and control crossfeeds, uncorrected for the digital nature of the controller. The other is for the digital system using optimal feedback gains and control crossfeeds calculated using Eqs. (12) and (13). At $\tau = 0.1$ sec (Fig. 4), the two cases are close to the desired response (Fig. 3) although the optimal digital controller is closer. At $\tau = 1.0$ sec, the response of the optimal digital system differs noticeably from

Table 1 Variation of feedback gains with cycling time—$\theta = 0$

	Cycling time Continuous-time	10^{-6} sec	10^{-1} sec	1 sec
δ_a/p	0.1533	0.1533	0.0892	−0.0669
δ_a/ϕ	0.0021	0.0021	−0.0020	−0.0153
δ_a/r	0.1180	0.1180	0.3296	0.9770
δ_a/β	−4.644	−4.644	−4.518	−3.187
δ_r/p	−0.0280	−0.0280	−0.0858	−0.1181
δ_r/ϕ	0	0	−0.0048	−0.0195
δ_r/r	0.3477	0.3477	0.5983	1.204
δ_r/β	−5.688	−5.688	−5.425	−0.990

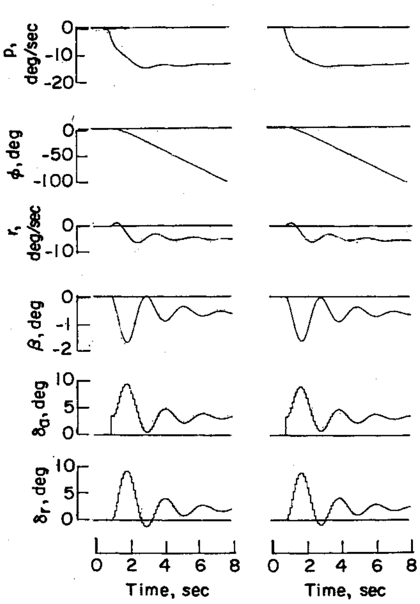
Table 2 Variation of control crossfeeds with cycling time—$\theta = 0$

	Cycling time Continuous-time	10^{-6} sec	10^{-1} sec	1 sec
δ_a/δ_{a_p}	1.000	1.000	0.982	1.253
δ_a/δ_{r_p}	0	0	−0.020	−0.357
δ_r/δ_{a_p}	0	0	0.076	1.326
δ_r/δ_{r_p}	1.000	1.000	0.931	−0.298



a) Unaugumented b) Continuous-time control

Fig. 3 Vehicle responses to a 3.67° step aileron input, a) continuous-time gains, b) optimal digital controller gains.



a) Continuous-time controller gains b) Optimal digital controller gains

Fig. 4 Vehicle responses to a 3.67° step aileron input using a digital controller with $\tau = 0.1$ sec, a) continuous-time controller gains, b) optimal digital controller gains.

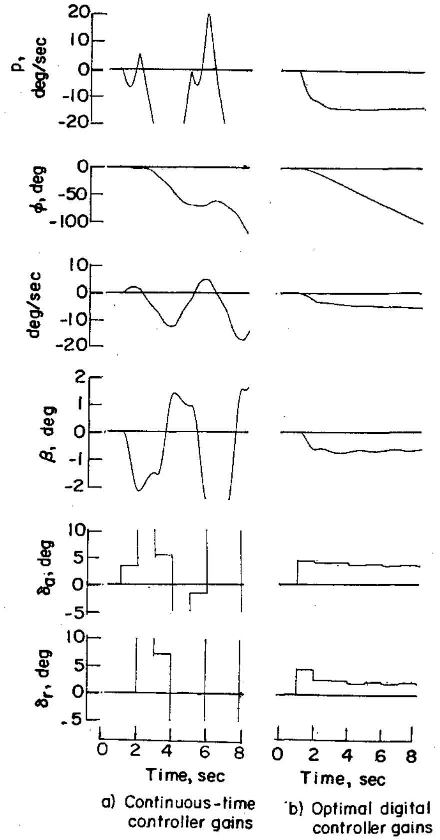


Fig. 5 Vehicle responses to a 3.67° step aileron input using a digital controller with $\tau = 1.0$ sec.

that of the analog system. Even though the responses using the optimal digital controller do not exactly match those of the analog controller for this case, the digital system is the optimal one. Indeed, when the digital controller was used with the analog systems gains, uncorrected for the discrete nature of the controller, the system was unstable at $\tau = 1.0$ sec.

Characteristics of Digital Control with Computational Delay

Computations have also been made including computation delay. The computational delay θ has a large effect on the controller feedback and crossfeed gains. However, for $\tau = 0.1$ sec and for values of θ from 0 to 0.04 sec, the closed-loop response of the vehicle to a step aileron input δ_{a_p} is indistinguishable from the no delay case on Fig. 4. But, for values of $\theta > 0.06$ sec, the system is unstable. This instability is a consequence of the rationale underlying the controller design. This is not a general result for all digital design processes. All other computational results indicate that the instability occurs at $\theta/\tau \cong 0.5$ regardless of τ. This is demonstrated for $\tau = 0.1$ sec by the z-plane poles presented in Table 3 for values of θ between 0 and 0.09 sec. The criterion for stability of the system in terms of the z-plane poles is that all z-plane poles have a magnitude less than unity.[3] This is true for $\theta/\tau < 0.5$ but at $\theta/\tau \cong 0.5$, the controller introduced z-plane poles of magnitude $\cong$ unity which became larger as θ/τ was increased.

Table 3 Variation of closed-loop z-plane poles with computation time at $\tau = 0.1$ sec

θ sec	Roll mode	Spiral mode	Dutch roll mode	Controller mode	Controller mode
0	0.8607	0.9990	$0.9143 \pm 0.2796i$	0	0
0.01	0.8607	0.9990	$0.9143 \pm 0.2796i$	−0.1080	−0.1093
0.03	0.8607	0.9990	$0.9143 \pm 0.2796i$	−0.4167	−0.4218
0.05	0.8607	0.9990	$0.9143 \pm 0.2796i$	−0.9564	−0.9720
0.07	0.8607	0.9990	$0.9143 \pm 0.2796i$	−2.2665	−2.3020
0.09	0.8607	0.9990	$0.9143 \pm 0.2796i$	−8.7353	−8.8900

Conclusions

A procedure for the analytic design of digital flight control systems to realized preselected flying qualities requirements at a single flight condition has been presented and demonstrated. It involves determining an analog model of the aircraft that possesses the desired flying qualities and then determining the optimal digital control system that gives closed-loop time responses as close to the desired ones as possible. This procedure directly determines the structure of the digital controller and, therefore, shows the relative importance of providing individual feedbacks and control crossfeeds to account for the discrete nature of the system. Classical z-transform methods are not capable of providing this information directly.

The design process has been demonstrated by designing a digital control system for a lifting-body entry vehicle. The structure of the digital controller was found to vary considerably with controller cycling time and computation delay time producing feedback gains and control crossfeeds which are substantially different from those of the analog system for moderate cycling times (say about 0.7 sec). The design process produced closed-loop time responses which closely matched the analog model's responses for cycling times between 0 and 1 sec and for computational delay times which were less than half of the controller cycling time. For delay times in excess of half the controller cycling time, the closed-loop system was unstable as shown by large z-plane poles introduced by the controller.

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